**Roanoke Valley Governor’s School for Science and Technology
Advanced Placement Accelerated Laboratory Calculus BC Competency List**

(Last updated: June 2022)

AP Accelerated Calculus BC provides motivated and talented students a unique opportunity to cover the AB and BC calculus content in one year. The major themes are limits, continuity, derivatives, optimization, related rates, integration techniques, series and approximation, parametric and polar functions, multivariable functions, differential equations, and real world modeling of scientific phenomena. Students are expected to obtain a qualifying score of 3, 4, or 5 on the AP Calculus BC exam at the end of this course if they take the Advanced Placement exam.

Students have the option of dual enrolling through Virginia Western community College. They may earn a total of 8 credits by successfully completing both semesters of this course. First semester correlates with Virginia Western’s course: MTH 263 (4 credits) and second semester correlates with Virginia Western’s course: MTH 264 (4 credits).

This course is taught using best practices in gifted education. Each competency is aligned with Hockett’s five principles of gifted education:

**Gifted Education Principles:**( Hockett, J.A. (2009) “Curriculum for Highly Able Learners That Conforms to General Education and Gifted Education Quality Indicators.” *Journal of Education for the Gifted***. Vol. 32, No. 3, p. 394-440)**

1. High-quality curriculum for gifted learners uses a conceptual approach to organize or explore content that is discipline based and integrative.
2. High-quality curriculum for gifted learners pursues advanced levels of understanding beyond the general education curriculum through abstraction, depth, breadth, and complexity.
3. High-quality curriculum for gifted learners asks students to use processes and materials that approximate those of an expert, disciplinarian, or practicing professional.
4. High-quality curriculum for gifted learners emphasizes problems, products, and performances that are true to life, and outcomes that are transformational.
5. High-quality curriculum for gifted learners is flexible enough to accommodate self-directed learning fueled by student interests, adjustments for pacing, and variety.

*External standards from Advanced Placement Calculus BC were referenced when reviewing these competencies. To the right of each Enabling Objective is notation indicating alignment with external standards and a relative priority/proficiency rating from A (highest) to D (lowest).*

COMPETENCY I

**Define and apply the properties of limits and continuity of functions.**

*Enabling Objectives:*

|  |  |  |
| --- | --- | --- |
| * 1. Define limits and use limit notation.
 | 1.2 | A |
| * 1. Estimate limit values from graphs.
 | 1.3 | A |
| * 1. Estimate limit values from tables.
 | 1.4 | A |
| * 1. Determine limits using algebraic properties of limits
 | 1.5 | A |
| * 1. Determine limits using algebraic manipulation.
 | 1.6 | A |
| * 1. Select an appropriate procedure for determining limits.
 | 1.7 | A |
| * 1. Determine limits using the Squeeze/Sandwich Theorem.
 | 1.8 | C |
| * 1. Connect multiple representations of limits.
 | 1.9 | B |
| * 1. Explore types of discontinuities as removable or nonremovable.
 | 1.10 | B |
| * 1. Definite continuity at a point using limits.
 | 1.11 | B |
| * 1. Confirm continuity over an interval.
 | 1.12 | B |
| * 1. Remove discontinuities through the use of an extended function, determined by the limit.
 | 1.13 | B |
| * 1. Connect infinite limits and vertical asymptotes.
 | 1.14 | A |
| * 1. Connect limits at infinity with horizontal asymptotes.
 | 1.15 | A |
| * 1. Extend the concept of continuity through the use of Intermediate Value Theorem.
 | 1.16 | B |

COMPENTENCY II

**Define differentiation and apply basic derivative rules.**

*Enabling Objectives:*

|  |  |  |
| --- | --- | --- |
| 1. Define average and instantaneous rates of change a point.
 | 2.1 | B |
| 1. Define the derivative of a function and use derivative notation.
 | 2.2 | A |
| 1. Estimate derivatives of a function at a point using a table of values or a graph.
 | 2.3 | C |
| 1. Connect differentiability and continuity.
 | 2.4 | B |
| 1. Determine when derivatives do and do not exist.
 | 2.5 | A |
| 1. Apply the power rule.
 | 2.6 | A |
| 1. Apply the constant, sum, difference, and constant multiple rules.
 | 2.7 | A |
| 1. Determine the derivatives of sinx, cosx, ex, and lnx.
 | 2.8 | A |
| 1. Apply the product rule.
 | 2.9 | A |
| 1. Apply the quotient rule.
 | 2.10 | A |
| 1. Find derivatives of tanx, cotx, sec, and cscx.
 | 2.11 | A |

COMPETENCY III

**Apply advanced differentiation techniques.**

*Enabling Objectives:*

|  |  |  |
| --- | --- | --- |
| 1. Apply the chain rule to derivatives of composite functions.
 | 3.1 | A |
| 1. Calculate higher order derivatives.
 | 3.6 | A |
| 1. Use implicit differentiation to find first and second derivatives.
 | 3.2 | A |
| 1. Differentiate inverse functions.
 | 3.3 | B |
| 1. Differentiate inverse trigonometric functions.
 | 3.4 | B |
| 1. Apply the technique of logarithmic differentiation.
 | Exceeds standard | C |
| 1. Select procedures for calculating derivatives.
 | 3.5 | A |

COMPETENCY IV

**Apply the concept of a derivative to contextual situations.**

*Enabling Objectives:*

|  |  |  |
| --- | --- | --- |
| 1. Interpret the meaning of the derivative in context as a rate of change.
 | 4.1 | A |
| 1. Connect the relationship between a function and its first and second derivative to position, velocity, and acceleration in the context of straight-line motion.
 | 4.2 | B |
| 1. Interpret rates of change in applied contexts other than motion.
 | 4.3 | C |
| 1. Apply implicit differentiation to related rates.
 | 4.4 | B |
| 1. Solve related rate problems.
 | 4.5 | B |
| 1. Approximate values of a function using local linearity and linearization.
 | 4.6 | A |
| 1. Use L’hospital’s rule to determine limits of indeterminate forms.
 | 4.7 | B |

#### COMPETENCY V

**Use differentiation techniques to analyze the behavior of functions and relations.**

#### *Enabling Objectives*

|  |  |  |
| --- | --- | --- |
| * 1. Given a function that is continuous over the closed interval [a,b] and differentiable on the (a,b), find the value of “c” that satisfies the Mean Value Theorem.
 | 5.1 | A |
| * 1. Apply Mean Value Theorem to graphical and tabular contexts of problems.
 | 5.1 | A |
| * 1. Apply the Extreme Value Theorem to functions over closed intervals.
 | 5.2 | B |
| * 1. Identify and distinguish between global and local extrema both graphically and analytically.
 | 5.2 | A |
| * 1. Identify critical and hypercritical values for a function both analytically and graphically.
 | 5.2 | A |
| * 1. Use the first derivative test to identify local extrema.
 | 5.4 | A |
| * 1. Use the second derivative test to identify local extrema.
 | 5.7 | B |
| * 1. Determine the intervals on which a function is increasing, decreasing or constant.
 | 5.3 | A |
| * 1. Determine the concavity intervals of a function and points of inflection.
 | 5.6 | A |
| * 1. Sketch the graph of a function and its corresponding derivatives.
 | 5.8 | B |
| * 1. Connect a function to its first derivative and its second derivative.
 | 5.8 | B |
| * 1. Use derivatives to solve optimization problems.
 | 5.11 | C |
| * 1. Analyze behaviors of implicitly defined relationships (such as conics) for horizontal and vertical tangents, etc.
 | 5.12 | C |

#### COMPETENCY VI

**Integration and accumulation of change**

#### *Enabling Objectives*

|  |  |  |
| --- | --- | --- |
| 1. Explore accumulations of change
 | 6.1 | B |
| 1. Approximate areas through Riemann and Trapezoidal Sums
 | 6.2 | A |
| 1. Define a definite integral as an infinite Riemann Sum
 | 6.3 | B |
| 1. Apply the Fundamental Theorem of Calculus to definite integrals and extend it to accumulation functions
 | 6.4, 6.7 | A |
| 1. Interpret the behavior of accumulation functions involving area
 | 6.5 | B |
| 1. Apply properties of definite integrals
 | 6.6 | A |
| 1. Find antiderivatives for definite and indefinite integrals. Apply basic rules and notation
 | 6.8 | A |
| 1. Integrate using substitution
 | 6.9 | A |
| 1. Integrate using long division and completing the square
 | 6.10 | B |
| 1. Integrate using integration by parts
 | 6.11 | A |
| 1. Integrate using partial fraction decomposition
 | 6.12 | A |
| 1. Evaluate improper integrals
 | 6.13 | B |
| 1. Select an appropriate technique of integration
 | 6.14 | A |

#### COMPETENCY VII

**Explore the basic foundations of differential equations and solve them.**

#### *Enabling Objectives*

|  |  |  |
| --- | --- | --- |
| * 1. Model situations with differential equations
 | 7.1 | C |
| 1. Verify solutions for differential equations.
 | 7.2 | A |
| 1. Sketch slope fields
 | 7.3 | A |
| 1. Interpret a slope field and how it relates to solutions to differential equations.
 | 7.4 | B |
| 1. Approximate solutions using Euler’s Method.
 | 7.5 | B |
| 1. Find general solutions using separation of variables.
 | 7.6 | A |
| 1. Find particular solutions using initial conditions and separation of variables.
 | 7.7 | A |
| 1. Derive exponential models using differential equations.
 | 7.8 | C |
| 1. Model logistic growth with differential equations.
 | 7.9 | C |

COMPETENCY VIII

**Identify the derivative as a function that describes properties of an original equation graphically.**

#### *Enabling Objectives*

|  |  |  |
| --- | --- | --- |
| * 1. Find the average value of a function on an interval (Mean Value Theorem for Integrals)
 | 8.1 | B |
| 1. Connect position, velocity, and acceleration of functions using integrals.
 | 8.2 | B |
| 1. Use accumulation functions and definite integrals in applied contexts.
 | 8.3 | B |
| 1. Find the area between curves expressed as functions of x.
 | 8.4 | A |
| 1. Find the area between curves expressed as functions of y.
 | 8.5 | A |
| 1. Find the area between curves that intersect at more than two points.
 | 8.6 | A |
| 1. Find the volume of a solid using cross sections of squares, rectangles, triangles, and semicircles.
 | 8.7, 8.8 | A |
| 1. Find the volume of a solid by revolving a region about the x or y- axes using the disc method.
 | 8.9 | A |
| 1. Find the volume of a solid by revolving a region about other horizontal of vertical axes using the disc method.
 | 8.10 | B |
| 1. Find the volume of a solid by revolving a region about the x or y- axes using the washer method.
 | 8.11 | A |
| 1. Find the volume of a solid by revolving a region about other horizontal of vertical axes using the washer method.
 | 8.12 | B |
| 1. Find the arc length of a smooth, planar curve. Extend this concept to the distance travelled.
 | 8.13 | C |

#### COMPETENCY IX

**Parametric equation, polar coordinates, and vector-valued functions**

#### *Enabling Objectives*

|  |  |  |
| --- | --- | --- |
| * 1. Define and differentiate parametric equations.
 | 9.1 | A |
| 1. Find second derivatives of parametric equations.
 | 9.2 | B |
| 1. Find arc length of given parametric equations.
 | 9.3 | B |
| 1. Define and differentiate vector-valued functions.
 | 9.4 | A |
| 1. Integrate vector-valued functions.
 | 9.5 | A |
| 1. Solve motion problems using parametric and vector-valued functions.
 | 9.6 | B |
| 1. Define polar coordinates and differentiate in polar form.
 | 9.7 | A |
| 1. Find the area of a polar region or the area bounded by a single polar curve.
 | 9.8 | B |
| 1. Find the area of the region bounded by two polar curves.
 | 9.9 | B |

#### COMPETENCY X

**Determine the convergence of an infinite sequence and series.**

#### *Enabling Objectives*

|  |  |  |
| --- | --- | --- |
| * 1. Define convergent and divergent infinite series
 | 10.1 | A |
| 1. Determine whether an infinite Geometric series converges and what it converges to.
 | 10.2 | A |
| 1. Apply the nth term test for divergence appropriately to infinite series.
 | 10.3 | A |
| 1. Determine the convergence of a telescoping series. If it converges, find its sum.
 | 10.1 | B |
| 1. Apply the limit and direct comparison tests appropriately to an infinite series.
 | 10.6 | A |
| 1. Determine the convergence of a p-series.
 | 10.5 | B |
| 1. Apply the integral test to appropriately to determine the convergence of an infinite series.
 | 10.4 | A |
| 1. Use the alternating series test to assess the convergence of an alternating series.
 | 10.7 | A |
| 1. Approximate the sum of an alternating series and determine its maximum value of its error using the first neglected term.
 | 10.10 | B |
| 1. Apply the ratio test appropriately to an infinite series.
 | 10.8 | A |
| 1. Apply the root test appropriately to an infinite series.
 | Exceeds standard | C |
| 1. Determine absolute or conditional convergence of an infinite series.
 | 10.9 | B |
| 1. Write a Taylor or Maclaurin polynomial for a particular function.
 | 10.11 | A |
| 1. Determine the error resulting from using a Taylor polynomial to approximate a value using the LaGrange Error Bound.
 | 10.12 | C |
| 1. Define a power series.
 | 10.14 | A |
| 1. Determine an interval and radius of convergence for a power series.
 | 10.13 | B |
| 1. Derive the Taylor series for sinx, cosx, ex, and
 | 10.14 | A |
| 1. Perform operations on Taylor series.
 | 10.15 | B |